The Design, Development and Evaluation of an Adaptive Learning Domain Model for Post-Primary Mathematics

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Abstract

This paper is based on a research project that involved the design, development and evaluation of a number of digital artefacts used to represent a domain model, which is a core component of adaptive learning systems. A domain model is an abstraction used to hierarchically represent concepts, and the relationships between concepts, in a particular subject domain. The domain, which was the focus of the research project, is Junior Cycle Mathematics in Ireland (age 12 to 15). The research methodology was an exploratory case study, and the evaluation utilised a mixed-methods research design. Mathematics teachers were surveyed (n=36), and key informants chosen from the fields of mathematics education and adaptive learning were interviewed (n=9). The results suggest that there is a strongly held belief among teachers and educationalists that teaching mathematics in a connected way, by focussing on the relationships between concepts, is of fundamental importance for optimal learning in the subject. The research findings also suggest that an adaptive learning system with a core domain model could support this focus on relationships between concepts, and ultimately enhance the teaching and learning of mathematics.

Practitioner Notes

What is already known about this topic

- A domain model is a fundamental component of adaptive learning systems and courseware.
- The adaptive hypermedia research community have created domain model authoring tools.

What this paper adds

- Procedures for decomposing coarse-grained learning outcomes (LOs) into fine-grained LOs, concepts, and topics for domain models.
- Evidence that mathematics teachers and educationalists believe it is extremely important to emphasise connections between concepts in the teaching and learning of mathematics.

Implications for practice/policy

- Research is required into examining pedagogical strategies for the teaching and learning of post-primary mathematics in a connected way.
- Research and development, and evaluation of, adaptive learning technology for the teaching and learning of post-primary mathematics is desirable.

Introduction

The research project outlined in this paper makes the case for using adaptive learning technologies in post-primary mathematics. The literature review cites evidence that these technologies have affordances that result in more effective learning for students. More specifically, this research concerned itself with the domain model layer of a typical adaptive learning architecture. The diagram below shows one such design: the Adaptive Hypermedia Application Model, better known as AHAM (Figure 1). This representation illustrates the essential components of an adaptive

learning architecture: the Domain Model, the User Model and the Teaching Model (De Bra, Houben, & Wu, 1999).
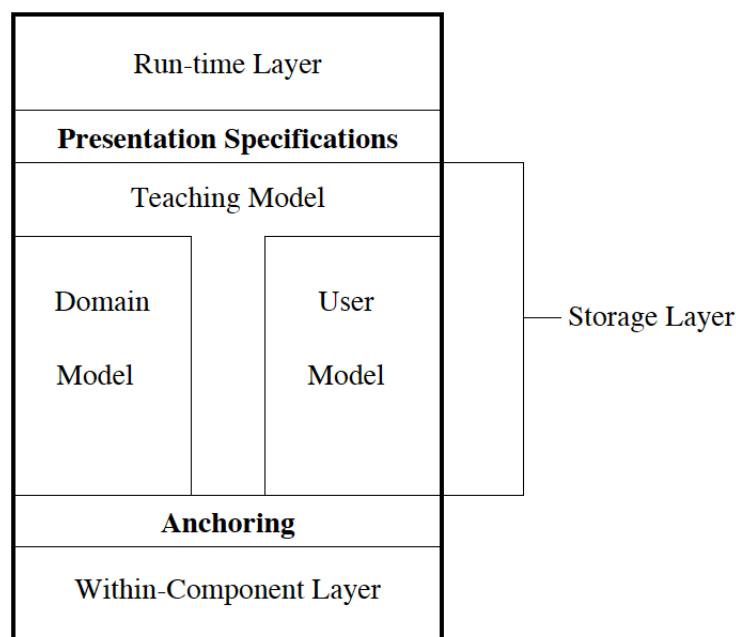

Figure 1: The AHAM Model

A domain model can be defined as a semantic structure of concepts and the relationships between these concepts (Aroyo, De Bra, Houben, & Vdovjak, 2004). In the domain model in Figure 2, an overlay user model is used to represent the user's knowledge of the various concepts in the domain model as a number (scalar) from 0 to 10 (Brusilovsky & Millán, 2007).
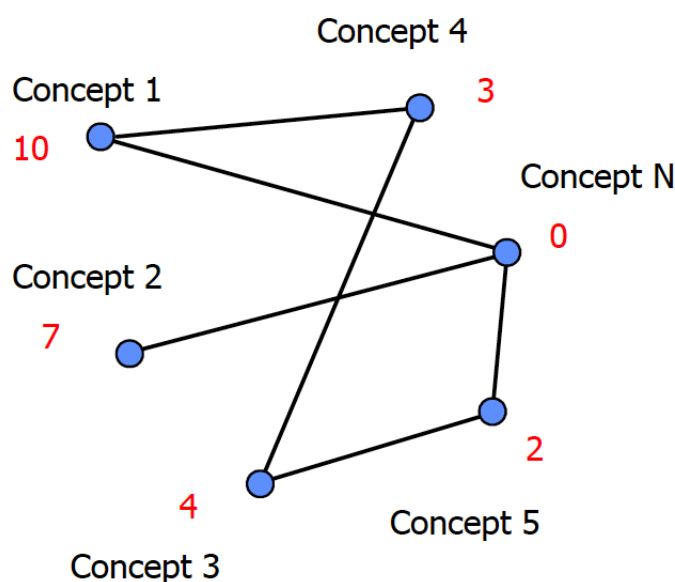

Figure 2: Domain Model with Overlay User Model

Connections are encouraged in the new Junior Cycle Mathematics syllabus, which states "students should be able to make connections within strands and between strands…" (NCCA, 2017). However, a report from the Chief Examiner for Junior Certificate Mathematics noted "candidates had great difficulty when required to make connections between a function and its graph in Paper 1, Question 13" (State Examinations Commission, 2016). Since a domain model can be used to represent relationships or connections between concepts, it may help students to make these connections. The new syllabus, published by the National Council for Curriculum and Assessment

(NCCA), will be implemented in all post-primary schools in Ireland from September 2018. It should be noted that a user's knowledge of concepts, denoted by scalars, cannot be stored in a user model if the domain model that represents the concepts does not exist. Therefore, the first step in creating an adaptive learning system is to build a domain model. However, there is evidence to suggest that the authoring of domain models, which are based on fine-grained concepts, is a difficult process for teachers to understand and perfect; it appears that they are more used to thinking about a domain in terms of coarse-grained topics (Sosnovsky & Brusilovsky, 2015). This leads to my research question:

"How can an adaptive learning domain model, for post-primary mathematics, be designed, developed and evaluated?"

Since the author is an experienced teacher of Mathematics, working in the post-primary sector in Ireland (age 12 to 18), and because of the issue with connections outlined above, the subject domain used in this research project was the syllabus for Junior Cycle Mathematics. This syllabus is designed for students aged 12 to 15 years old. The overall aim of the research was to design, develop and evaluate a domain model using specific learning outcomes from the new syllabus (NCCA, 2017). It was intended that these learning outcomes could be used to create the fine-grained concepts required for a domain model. This overall aim may be broken down into the following objectives:

(a) Identify concepts in Co-ordinate Geometry of the Line, Patterns, and Functions (Strands GT.5, AF.1, and AF.7 respectively in the new syllabus) that have strong ontological connections and use them in the domain model.

(b) Design a domain model for the identified concepts by using this syllabus as the data source.

(c) Design and develop digital artefacts encapsulating and explicating the domain model.

(d) Evaluate the domain model embedded in these artefacts using professionals working in mathematics education and adaptive learning.

The background for the proposed research project is outlined in a comprehensive literature review that provides the reader with an understanding of how adaptive learning is modelled. The literature review also gives a historical perspective of the development of Adaptive Educational Hypermedia Systems (AEHS) and Intelligent Tutor Systems (ITS). The authoring systems that are required to build AEHS/ITS are also examined.

The design and development of three digital artefacts that encapsulate variations of a domain model are presented and discussed in this paper. The research design utilised an exploratory case study involving a survey of mathematics teachers (n=36) and in-depth semi-structured interviews with key informants. The key informants were mathematics education professionals (n=7) and adaptive learning experts (n=2).

The results from the survey of mathematics teachers are presented later. The themes, explored more deeply in the interviews with the key informants, are also presented and discussed. Finally, some conclusions are drawn, and areas for further research are suggested.

Literature Review
Background
Many researchers refer to the "one-size-fits-all" approach of serving static content electronically to users. Static content exhibits none of the benefits of interaction and personalisation offered by AEHS (Brusilovsky, 2001; Šimko, Barla, & Bieliková, 2010; De Bra, 2017). An AEHS is a Web-based system that adapts to the needs of different users by building a user model of their goals, preferences and knowledge. Personalised e-learning is achieved through the use of adaptive

systems (Brusilovsky, 2001). AEHS were originally developed between 1990 and 1996 by either taking existing ITS and adding hypermedia components or by taking existing educational hypermedia and adding adaptive features (Brusilovsky, 2003). For a system to be classified as an Adaptive Hypermedia System (AHS), it should satisfy three criteria: it should have hypertext or hypermedia, a user model, and the ability to adapt the hypermedia using this user model (Brusilovsky, 1996).

Adaptive Learning Models
Some of the adaptive learning authoring tools that have been built to create personalised e-learning activities are based on abstract designs or reference models (O'Donnell, Lawless, Sharp, & Wade, 2015). In the introduction, one such design AHAM was examined (Figure 1). To understand how AHAM works, it is necessary to explore some of its essential components: domain model, user model, and teaching model.

A domain model has already been defined as a semantic structure of concepts and the relationships between these concepts (Aroyo et al., 2004). Ahmad, Basir, & Hassanein, 2004 provide a more comprehensive definition of a domain model:

> The Domain Model (DM) is the abstract representations of the target subject area. It deals with the link relationships between the concepts and the decomposition of concepts in a structured hierarchy of sub-concepts and atomic information such as texts, images, sounds, and videos. (p. 928)

Šimko (2012) emphasises the importance of a domain model when he states that the "adaptation engine responsible for advanced functionality in the educational system relies on the domain model semantically describing subject domain."

In truly personalised systems, the user model can represent the user's knowledge, interests, goals, background, and individual traits. In ITS, the user model is known as the student model and represents mainly the user's knowledge of the subject or domain. Since user knowledge is the principal feature being modelled in the user model of an AEHS or ITS, adaptive learning systems often use the overlay model referred to in Figure 2 where the user's knowledge of the various concepts in the domain model is represented by a scalar (Brusilovsky & Millán, 2007). The importance of capturing knowledge for the user model was underlined by the results of a survey of academics on personalised eLearning in higher education. 55% of those surveyed stated that prior knowledge was the most important student characteristic on which to base personalisation (O'Donnell, Sharp, Wade, & O'Donnell, 2012).

The teaching model contains a set of pedagogical rules (O'Donnell, Sharp, Wade, & O'Donnell, 2012).. These rules are used by an *adaptive engine* to generate personalised content based on the learners' knowledge and performance, which is stored in the user model (Vassileva, Bontchev, Chavkova, & Mitev, 2009). An adaptive engine is the software that is used to construct and adapt content and links based on elements from the various models (Wu, Houben, & De Bra, 1998). In the case of AHAM, the adaptive engine uses the pedagogical rules to manipulate link anchors from the *anchoring* and to generate the *presentation specifications* (refer to Figure 1) for the personalised content (Wu, Houben, & De Bra, 1999). The teaching model is also known as the application model (Aroyo et al., 2004).

The Case for Adaptive Learning and ITS
Most of the eLearning course materials available today are oriented for a homogeneous audience of well-prepared and well-motivated students who have access to teachers. However, learners have very different goals, knowledge levels, and learning abilities. Surely there is an *a priori* case to

make for students to receive personalised content and a personalised order of presentation? The consequence of "one-size-fits-all" non-adaptive static content is that some students waste time by reading material that they already know, while others are presented with content that is beyond their current capabilities (Brusilovsky, Eklund, & Schwarz, 1998).

In a meta-analysis of 50 controlled evaluations of ITS, Kulik & Fletcher, 2016 describe evaluations that were carried out on four continents over the course of nearly three decades. They reported an average effect size of 0.66, meaning that the score of the average person in the experimental group was 0.66 standard deviations above the average person in the control group. Another meta-analysis involving 14,321 participants established an average effect size of 0.57. It discovered that the use of ITS was linked with significantly higher achievement outcomes than all other modes of instruction except small-group human tutoring and individual human tutoring (Ma, Adesope, Nesbit, & Liu, 2014; Coe, 2002).

Adaptive Authoring Tools

Given the *a priori* case for adaptive learning, and the evidence for the effectiveness of ITS, why is adaptive learning not more prevalent today? While authoring tools for adaptive learning have been created by academics working in universities, and by commercial organisations, it appears that the main barrier to the mainstream adoption of adaptive learning is the complexity of existing authoring tools (O'Donnell, Sharp, Wade, & O'Donnell, 2013).

In the academic sector, there are at least three strands of authoring tool development that this author has identified, and specific adaptive learning models underpin these strands. This development work occurred between 1996 and 2013, and to a large extent the three strands occurred concurrently.

The first authoring tools were developed by Peter Brusilovsky and his team at Pittsburgh University and led to the development of InterBook in 1996 (Brusilovsky et al., 1998). This was followed up with the release of KnowledgeTree two years later (Brusilovsky, 2004).

The second strand was based on the AHAM model and is associated with Professor Paul De Bra and his team of researchers in Technical University, Eindhoven (TU/e). He instigated the development of AHA! in 2003 and its development continued until 2007 (Wu et al., 1999; De Bra, 2007). A four-year European research project (GRAPPLE) built on the work of AHA! (GRAPPLE, 2011; Smits & De Bra, 2011; De Bra et al., 2013).

Dr Alexandra Cristea and her team in University of Warwick created the third strand of authoring tools. They developed a tool called My Online Teacher (MOT) between 2000 and 2007, which was based on an adaptive architecture called LAOS (Cristea, 2007).

A number of authoring tools for adaptive learning have emerged from the commercial sector in recent years. A comprehensive list of these tools, with accompanying descriptions, may be found in a PDF document available online and updated regularly (McIntosh, 2018). In the section 'Adaptive Learning Platforms', the adaptive learning products listed are a mixture of learning management systems (LMS) with adaptive features, adaptive platforms designed to be integrated with LMS, and adaptive learning courseware.

Summary

This research study was motivated by a number of issues. First, the effectiveness of AEHS and ITS has been acknowledged in numerous research studies (Kulik & Fletcher, 2016). Second, a domain model is an essential part of an AEHS (Vrablecova & Simko, 2016). Third, recent Junior Certificate/Junior Cycle Mathematics syllabuses do not comprehensively define the topics and concepts, nor do they highlight connections between concepts (Department of Education and Skills,

2016; NCCA, 2017). Fourth, existing hard copy and non-hyperlinked electronic versions of textbooks are by their very nature unsuitable for adaptive learning. Fifth, in recent years there has been a paradigm shift from whole class instruction to individualised learning. Adaptive learning can support this shift (Jenkins, Williams, Moyer, George, & Foster, n.d.). Finally, there is evidence to suggest that students find it difficult to make connections between concepts (State Examinations Commission, 2016).

Technical Implementation
Background
The new Junior Cycle Mathematics syllabus consists of four stands described as learning outcomes (LOs) (NCCA, 2017). This research project chose eight specific LOs from Co-ordinate Geometry of the Line, Patterns, and Functions for their interconnectedness. The first step in the design and development of the domain model artefacts involved the use of Microsoft Excel. This application was used to decompose the eight syllabus LOs into fine-grained LOs and concepts, and later into topics. The rationale for unpacking the LOs is that fine-grained domain models are required for precise adaptation (Sosnovsky & Brusilovsky, 2015). The second step was the creation of the actual artefacts using three software tools. These were the GAM Authoring Tool, Mindomo and Rhumbl (Craenen, 2017; Mindomo, 2018; Rhumbl, 2018).

Decomposition of Syllabus LOs into Finely Grained LOs, Concepts and Topics
Ahmad et al. (2004) state that the domain model deals with "…the decomposition of concepts in a structured hierarchy of sub-concepts and atomic information…" In this research project, the initial target objects for decomposition were LOs. This is because the syllabus specifies LOs rather than concepts or topics. The decomposition process executed in this research project can be described by three distinct phases.

Phase 1 – Unpacking the 8 Syllabus LOs to 45 Finely Grained LOs
An example of this unpacking process is illustrated by taking one of the syllabus LOs relating to Patterns, **AF.1a** (Figure 3) and unpacking it into the six LOs **AF 1a-1** to **AF 1a-6** (Figure 4).
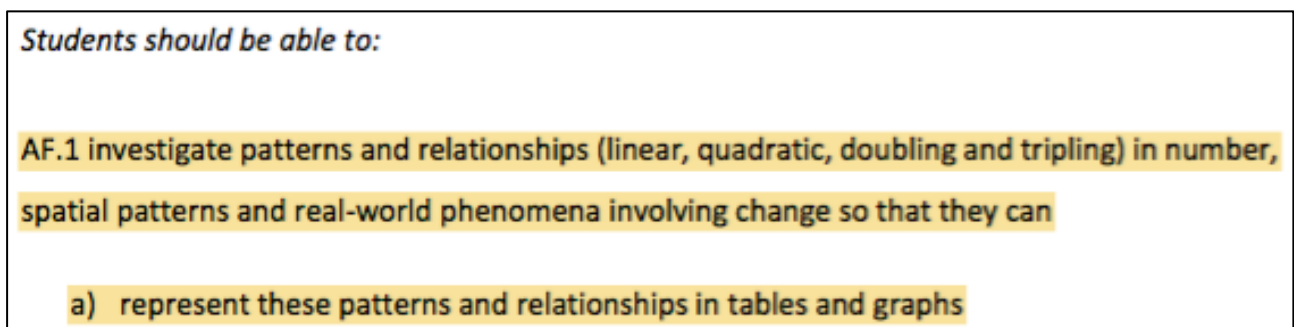

Students should be able to:

AF.1 investigate patterns and relationships (linear, quadratic, doubling and tripling) in number, spatial patterns and real-world phenomena involving change so that they can

a)   represent these patterns and relationships in tables and graphs

Figure 3: One Syllabus Learning Outcome: Patterns (AF.1a)

11	Strand	Outcome	Unpacked Learning Outcomes	Level
12	AF	1a-1	represent linear patterns and relationship in tables	O
13	AF	1a-2	represent linear patterns and relationships in graphs	O
14	AF	1a-3	represent quadratic patterns and relationships in tables	O
15	AF	1a-4	represent quadratic patterns and relationships in graphs	O
16	AF	1a-5	represent exponential patterns and relationships in tables	O
17	AF	1a-6	represent exponential patterns and relationships in graphs	O

Figure 4: Six Unpacked Learning Outcomes (AF 1a-1 to AF 1a-6)

Two further examples of unpacking a syllabus LO relate to Functions (Figure 5 and Figure 6) and Co-ordinate Geometry of the Line (Figure 7 and Figure 8)**.**

AF.7 investigate the concept of a function so that they can

a) demonstrate understanding of a function

b) represent and interpret functions in different ways – graphically (for x ∈ ℕ, ℤ, and ℝ,

Figure 5: One Syllabus Learning Outcome: Functions (AF.7b)

28	Strand	Outcome	Unpacked Learning Outcomes	Level
29	AF	7b-1	represent linear functions in tables - using x, f(x), y, domain, range, co-domain	O
30	AF	7b-2	represent linear functions graphically - using x, y, domain, range, co-domain	O
31				
32	AF	7b-4	represent quadratic functions in tables - using x, f(x), y, domain, range, co-domain	O
33	AF	7b-5	represent quadratic functions graphically - using x, y, domain, range, co-domain	O
34				
35	AF	7b-7	represent exponential functions in tables - using x, f(x), y, domain, range, co-domain	H
36	AF	7b-8	represent exponential functions graphically - using x, y, domain, range, co-domain	H

Figure 6: Six Unpacked Learning Outcomes (AF 7b-1 to AF 7b-8)

GT.5 investigate properties of points, lines and line segments in the co-ordinate plane so that they can

a) find and interpret: distance, midpoint, slope, point of intersection, and slopes of parallel **and perpendicular** lines

b) draw graphs of line segments and interpret such graphs in context, including discussing the rate of change (slope) and the y intercept

Figure 7: One Syllabus Learning Outcome: The Line (GT.5b)

Strand	Outcome 1	Outcome 2	Linear Learning Outcomes	Level
THE LINE - Distance, Slope, Equation				
GT	5b-1		draw graphs of line segments	O
GT	5b-2		extend line to find y intercept graphically	O
LINEAR PATTERNS & FUNCTIONS - Categorise, Interpret, Understand				
GT	5b-3		interpret linear functions/line segments in context, including discussing rate of change (slope) and y intercept	O

Figure 8: Three Unpacked Learning Outcomes (GT 5b-1 to GT 5b-3)

All 45 unpacked LOs can be seen in Figure 11. A colour scheme was used whereby similar representations of concepts have similar colours. For example, orange is used for 'Tables' and green is used for 'Graphs'. The reason for doing this was that it helped to build the hierarchies of concepts used in the Mindomo artefact and to make connections between topics and LOs in the Rhumbl Maps artefact.

Phase 2 – Unifying the Learning Outcomes for Patterns and Functions
An example of the unification process can be seen in Figure 9. The six LOs relating to Patterns (Figure 4) are paired up with six corresponding LOs relating to Functions (Figure 6). For example, there is now a single unified LO from **AF 1a-1** and **AF 7b-1** described as "represent linear patterns and functions in tables" (Figure 9).

15	Strand	Outcome 1	Outcome 2	Unified Learning Outcomes	Level
16	LINEAR PATTERNS & FUNCTIONS - Tables, Graphs				
17	AF	1a-1	7b-1	represent linear patterns and functions in tables	O
18	AF	1a-2	7b-2	represent linear patterns and functions graphically	O
19	QUADRATIC PATTERNS & FUNCTIONS - Tables, Graphs				
20	AF	1a-3	7b-4	represent quadratic patterns and functions in tables	O
21	AF	1a-4	7b-5	represent quadratic patterns and functions graphically	O
22	EXPONENTIAL PATTERNS & FUNCTIONS - Tables, Graphs				
23	AF	1a-5	7b-7	represent exponential patterns and functions in tables	H
24	AF	1a-6	7b-8	represent exponential patterns and functions graphically	H

Figure 9: Six Pairs of Unified Learning Outcomes (AF 1a-1 to AF 7b-8)

Phase 3 – Distilling the Learning Outcomes for The Line, Linear Patterns, Linear Functions
The distilling process involved removing any LOs that did not have the words 'linear' or 'line' from the 45 unpacked LOs created in Phase 1 (Figure 11).

Some of the 'Linear' LOs arrived at in Phase 3 can be seen by combining the three LOs in The Line and Linear Patterns & Functions (Figure 8) with the four LOs in Linear Patterns & Functions (Figure 9). This gives the seven 'Linear' LOs that can be seen in Figure 10.

Strand	Outcome	Linear Learning Outcomes	Level
THE LINE - Distance, Slope, Equation			
GT	5b-1	draw graphs of line segments	O
GT	5b-2	extend line to find y intercept graphically	O
LINEAR PATTERNS & FUNCTIONS - Tables, Graphs			
AF	1a-1	represent linear patterns in tables	O
AF	1a-2	represent linear patterns graphically	O
AF	7b-1	represent linear functions in tables	O
AF	7b-2	represent linear functions graphically	O
LINEAR PATTERNS & FUNCTIONS - Categorise, Interpret, Understand			
GT	5b-3	interpret linear functions/line segments in context, including discussing rate of change (slope) and y intercept	O

Figure 10: Seven Linear Learning Outcomes (GT 5b-1 to GT 5b-3)

Design and Development of the Domain Model Artefacts
The 45 unpacked LOs from Phase 1 (Figure 11) were used to create the Rhumbl Maps artefact (Figure 14 and Figure 15). The 33 unified LOs from Phase 2 (Figure 16) were used to build the Mindomo artefact (Figure 17). Finally, the 22 linear LOs (Figure 18) helped construct the GAM AT artefact (Figure 19 and Figure 20).

Rhumbl Maps Domain Model
Rhumbl Maps was chosen as a domain model authoring tool because it is described as being capable of authoring learning outcomes for adaptive learning (Rhumbl, 2018). It achieves this by linking LOs to topics using three purpose built Excel spreadsheet templates for LOs, topics, and a matrix of LOs/topics. In simple terms, a topic, e.g. 'Patterns and Relationships', corresponds to a coarse-grained LO, e.g. investigate patterns and relationships (Figure 3) and a set of concepts are encapsulated in a topic (Sosnovsky & Brusilovsky, 2015). Linking LOs to topics is a non-conventional approach to adaptive learning. The traditional approach of concept-based adaptation links concepts to concepts (Sosnovsky & Brusilovsky, 2015). The first spreadsheet was populated with the 45 unpacked LOs from Phase 1 (Figure 11).

name	short name	group
Instructions [Do not delete this row]: In each row, list the name and short name of your learning outcome. Specify the group that it belongs to. An outcome can belong to only one group. Your outcomes will be clustered by group, and the group name will appear on the map as a cluster label.		
name	short name	group
5a-1 find distance (slant), using Pythagoras theorem	5a-1	5a-1
5a-2 find distance (slant), using formula	5a-2	5a-2
5a-3 find and interpret slope, using rise and run	5a-3	5a-3
5a-4 find slope, using formula	5a-4	5a-4
5b-1 draw graphs of line segments	5b-1	5b-1
5b-2 extend line to find y intercept graphically	5b-2	5b-2
5b-3 interpret linear functions/line segments in context, including discussing rate of change (slope) and y intercept	5b-3	5b-3
5c-1 find the equation of a line in the form y = mx + c	5c-1	5c-1
5c-2 interpret the equation of a line in the form y = mx + c, including finding the slope	5c-2	5c-2
5c-3 interpret the equation of a line in the form y = mx + c, including finding the y intercept	5c-3	5c-3
5c-4 interpret the equation of a line in the form y = mx + c, including finding more points	5c-4	5c-4
1a-1 represent linear patterns and relationship in tables	1a-1	1a-1
1a-2 represent linear patterns and relationships in graphs	1a-2	1a-2
1a-3 represent quadratic patterns and relationships in tables	1a-3	1a-3
1a-4 represent quadratic patterns and relationships in graphs	1a-4	1a-4
1a-5 represent exponential patterns and relationships in tables	1a-5	1a-5
1a-6 represent exponential patterns and relationships in graphs	1a-6	1a-6
1b-1 write a generalised expression for linear patterns in words	1b-1	1b-1
1b-2 write a generalised expression for linear patterns algebraically	1b-2	1b-2
1b-3 write a generalised expression for quadratic patterns in words	1b-3	1b-3
1b-4 write a generalised expression for quadratic patterns algebraically	1b-4	1b-4
1b-5 write generalised expressions for exponential patterns in words	1b-5	1b-5
1b-6 write generalised expressions for exponential patterns algebraically	1b-6	1b-6
1c-1 categorise patterns as linear, non-linear, quadratic, and exponential	1c-1	1c-1
1c-2 find difference (d) and first term (a) for a linear pattern	1c-2	1c-2
1c-3 find first and second differences for a quadratic pattern	1c-3	1c-3
1c-4 Find ratio (r) and differences ratio (r) for an exponential pattern	1c-4	1c-4
7a demonstrate understanding of a function	7a	7a
7b-1 represent linear functions in tables	7b-1	7b-1
7b-2 represent linear functions graphically	7b-2	7b-2
7b-3 represent linear functions diagrammatically	7b-3	7b-3
7b-4 represent quadratic functions in tables	7b-4	7b-4
7b-5 represent quadratic functions graphically	7b-5	7b-5
7b-6 represent quadratic functions diagrammatically	7b-6	7b-6
7b-7 represent exponential functions in tables	7b-7	7b-7
7b-8 represent exponential functions graphically	7b-8	7b-8
7b-9 represent exponential functions diagrammatically	7b-9	7b-9
7b-10 represent linear functions in words	7b-10	7b-10
7b-11 represent linear functions algebraically	7b-11	7b-11
7b-12 represent quadratic functions in words	7b-12	7b-12
7b-13 represent quadratic functions algebraically	7b-13	7b-13
7b-14 represent exponential functions in words	7b-14	7b-14
7b-15 represent exponential functions algebraically	7b-15	7b-15
7d-1 interpret quadratic functions, including predicting the shape algebraically and identifying the turning point graphically	7d-1	7d-1
7d-2 interpret exponential functions, including connecting rapid increase with variable as exponent and significance of point (1,0)	7d-2	7d-2

Figure 11: Rhumbl Spreadsheet (45 Unpacked Learning Outcomes)

The second spreadsheet was populated with 11 topics formulated from the coarse-grained LOs (Figure 12). The first five topics relate to Figure 9 and Figure 10. The extra six topics ('Tables', 'Graphs', etc.) allowed for a greater number of connections between topics and LOs, to allow for more effective adaptation (Sosnovsky & Brusilovsky, 2015).

name	short name	group
The Line	Line	GT-1
Linear Patterns & Functions	Linear	AF-1
Quadratic Patterns & Functions	Quadratic	AF-2
Exponential Patterns & Functions	Exponential	AF-3
Categorise/Interpret Patterns & Functions	Categorise	AF-4
Tables	Tables	AF-5
Graphs	Graphs	AF-6
Venn Diagrams	Venn Diagrams	AF-7
Algebraic Expressions (Linear)	Algebra-Linear	AF-8
Algebraic Expressions (Quadratic)	Algebra-Quadratic	AF-9
Algebraic Expressions (Exponential)	Algebra-Exponential	AF-10

Figure 12: Rhumbl Spreadsheet (Topics)

The connections were created using a third spreadsheet, a matrix of topics and LOs (Figure 13). The digit '1' in a cell indicates a connection. For example, LO 1a-2 is connected to three topics.

	A	M	N	O	P	Q	R	S	T
	Instructions (Do not delete or modify this cell): In column A, list the name of topic. It must exactly match what you previousl the entities__topic worksheet, so we recommend you use the Excel = formul reference the topic name. In row 1, list the the outcomes. Again, it needs to exactly match what is listed entities_learning outcome worksheet. Then, put a "1" if a topic addresses an outcome. E.g. put a "1" in Row 3, Colu 2 addresses Learning Outcome 1	1a-1 represent linear patterns and relationship in tables	1a-2 represent linear patterns and relationship s in graphs	1a-3 represent quadratic patterns and relationship s in tables	1a-4 represent quadratic patterns and relationship s in graphs	1a-5 represent exponential patterns and relationship s in tables	1a-6 represent exponential patterns and relationship s in graphs	1b-1 write a generalised expression for linear patterns in words	1b-2 write a generalised expression for linear patterns algebraically
2	The Line		1						
3	Linear Patterns & Functions	1	1					1	1
4	Quadratic Patterns & Functions			1	1				
5	Exponential Patterns & Functions					1	1		
6	Categorise/Interpret Patterns & Functions								
7	Tables	1		1		1			
8	Graphs		1		1		1		
9	Venn Diagrams								
10	Algebraic Expressions (Linear)							1	1
11	Algebraic Expressions (Quadratic)								
12	Algebraic Expressions (Exponential)								

Figure 13: Rhumbl Spreadsheet (Matrix)

This Excel spreadsheet matrix of topics and LOs is used by Rhumbl to generate two different views. The 'Topic View' is generated when the user clicks on a topic, e.g. Tables (Figure 14).

Figure 14: Rhumbl Map Domain Model (Topic View)

The second display type, the 'Outcome View', is generated when the user clicks on a learning outcome, e.g. 7b-2 (Figure 15).

Figure 15: Rhumbl Map Domain Model (Outcome View)

Mindomo Domain Model

To complete Phase 2, all pairs of LOs for patterns and functions from Phase 1 were unified. There were 12 such pairs, and with each pair counting as one LO, the result was 33 LOs (Figure 16). The reason for doing this is that one of the objectives of this research project was to connect Patterns (Strand AF.1) and Functions (Strand AF.7), which have strong ontological connections, so that these connections could be represented in a domain model. These 33 LOs were used to create the

Mindomo domain model artefact, which emphasises the relationship between the concepts patterns and functions (Figure 17).

E	F	G	H	I	J
	Strand	Outcome 1	Outcome 2	Unified Learning Outcomes	Level
	THE LINE - Distance, Slope, Equation				
1	GT	5b-1		draw graphs of line segments	O
2	GT	5a-1		find distance (slant), using Pythagoras theorem	O
3	GT	5a-2		find distance (slant), using formula	O
4	GT	5a-3		find and interpret slope, using rise and run	O
5	GT	5a-4		find slope, using formula	O
6	GT	5b-2		extend line to find y intercept graphically	O
7	GT	5c-1		find the equation of a line in the form y = mx + c	O
8	GT	5c-2		interpret the equation of a line in the form y = mx + c, including finding the slope	O
9	GT	5c-3		interpret the equation of a line in the form y = mx + c, including finding the y intercept	O
10	GT	5c-4		interpret the equation of a line in the form y = mx + c, including finding more points	O
	LINEAR PATTERNS & FUNCTIONS - Tables, Word Formulas, Algebra Formulas, Graphs, Venn Diagrams				
11	AF	1a-1	7b-1	represent linear patterns and functions in tables	O
12	AF	1b-1	7b-10	represent linear patterns and functions in words	O
13	AF	1b-2	7b-11	represent linear patterns and functions algebraically	O
14	AF	1a-2	7b-2	represent linear patterns and functions graphically	O
15	AF		7b-3	represent linear functions diagrammatically	O
	QUADRATIC PATTERNS & FUNCTIONS - Tables, Word Formulas, Algebra Formulas, Graphs, Venn Diagrams				
16	AF	1a-3	7b-4	represent quadratic patterns and functions in tables	O
17	AF	1b-3	7b-12	represent quadratic patterns and functions in words	O
18	AF	1b-4	7b-13	represent quadratic patterns and functions algebraically	O
19	AF	1a-4	7b-5	represent quadratic patterns and functions graphically	O
20	AF		7b-6	represent quadratic functions diagrammatically	O
	EXPONENTIAL PATTERNS & FUNCTIONS - Tables, Word Formulas, Algebra Formulas, Graphs, Venn Diagrams				
21	AF	1a-5	7b-7	represent exponential patterns and functions in tables	H
22	AF	1b-5	7b-14	represent exponential patterns and functions in words	H
23	AF	1b-6	7b-15	represent exponential patterns and functions algebraically	H
24	AF	1a-6	7b-8	represent exponential patterns and functions graphically	H
25	AF		7b-9	represent exponential functions diagrammatically	H
	LINEAR, QUADRATIC, EXPONENTIAL PATTERNS & FUNCTIONS - Categorise, Interpret, Understand				
26	AF	1c-1		categorise patterns as linear, non-linear, quadratic, and exponential	O
27	AF	1c-2		find difference (d) and first term (a) for a linear pattern	O
28	AF	1c-3		find first and second differences for a quadratic pattern	O
29	AF	1c-4		Find ratio (r) and differences ratio (r) for an exponential pattern	H
30	GT	5b-3		interpret linear functions/line segments in context, including discussing rate of change (slope) and y intercept	O
31	AF	7d-1		interpret quadratic functions, including predicting the shape algebraically and identifying the turning point graphically	H
32	AF	7d-2		interpret exponential functions, including connecting rapid increase with variable as exponent and significance of point (1,0)	H
33	AF	7a		demonstrate understanding of a function	O

Figure 16: The 33 Unified Learning Outcomes (Phase 2)
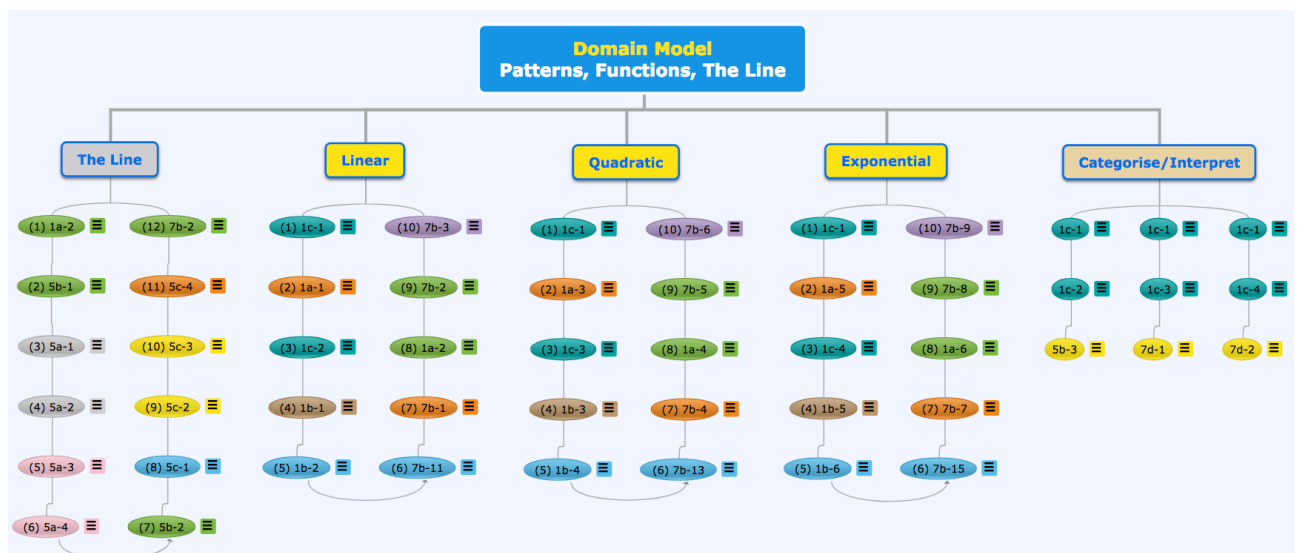
Figure 17: Mindomo Domain Model

GAM AT Domain Model

The GAM AT software used to build the third and final domain model artefact was developed in 2017 (Craenen, 2017). The development of this tool built on the work that created the Adaptive Learning Authoring Tool (ALAT) (Boereboom, 2016; De Bra et al., 2016). ALAT in turn was an evolution of the GRAPPLE Authoring Tool (GAT) (De Bra et al., 2013; GRAPPLE, 2011).

This domain model was created to represent the rich connections that exist between the concepts encapsulated in the strand topics Co-ordinate Geometry of the Line, Linear Patterns, and Linear Functions. GAM AT uses a hierarchy of fine-grained concepts. The completion of the Phase 3 process resulted in 22 LOs (Figure 18) that could then be represented as the 22 single word concepts required by the GAM AT functionality (Figure 19 and Figure 20).

	Strand	Outcome	Linear Learning Outcomes	Level
	THE LINE - Distance, Slope, Equation			
1	GT	5b-1	draw graphs of line segments	O
2	GT	5a-1	find distance (slant), using Pythagoras theorem	O
3	GT	5a-2	find distance (slant), using formula	O
4	GT	5a-3	find and interpret slope, using rise and run	O
5	GT	5a-4	find slope, using formula	O
6	GT	5b-2	extend line to find y intercept graphically	O
7	GT	5c-1	find the equation of a line in the form y = mx + c	O
8	GT	5c-2	interpret the equation of a line in the form y = mx + c, including finding the slope	O
9	GT	5c-3	interpret the equation of a line in the form y = mx + c, including finding the y intercept	O
10	GT	5c-4	interpret the equation of a line in the form y = mx + c, including finding more points	O
	LINEAR PATTERNS & FUNCTIONS - Tables, Word Formulas, Algebra Formulas, Graphs, Venn Diagrams			
11	AF	1a-1	represent linear patterns in tables	O
12	AF	1b-1	represent linear patterns in words	O
13	AF	1b-2	represent linear patterns algebraically	O
14	AF	1a-2	represent linear patterns graphically	O
15	AF	7b-1	represent linear functions in tables	O
16	AF	7b-10	represent linear functions in words	O
17	AF	7b-11	represent linear functions algebraically	O
18	AF	7b-2	represent linear functions graphically	O
19	AF	7b-3	represent linear functions diagrammatically	O
	LINEAR, QUADRATIC, EXPONENTIAL PATTERNS & FUNCTIONS - Categorise, Interpret, Understand			
20	AF	1c-2	find difference (d) and first term (a) for a linear pattern	O
21	GT	5b-3	interpret linear functions/line segments in context, including discussing rate of change (slope) and y intercept	O
22	AF	7a	demonstrate understanding of a function	O

Figure 18: The 22 Linear Learning Outcomes (Phase 3)

Concept hierarchy ⓘ

Linear
Pattern
TableTn
DifferenceD
WordExpression
AlgExpressionTn
FunctionFx
TableFx
GraphXY
PointsXY
VennDomainX
VennRangeY
TheLine
TwoPointsXY
LineSegment
DistancePythag
DistanceFormula
SlopeRiseRun
SlopeFormula
LineSegmentExt
EquationMxc
SlopeM
YinterceptC

Figure 19: GAM AT Domain Model (List View)
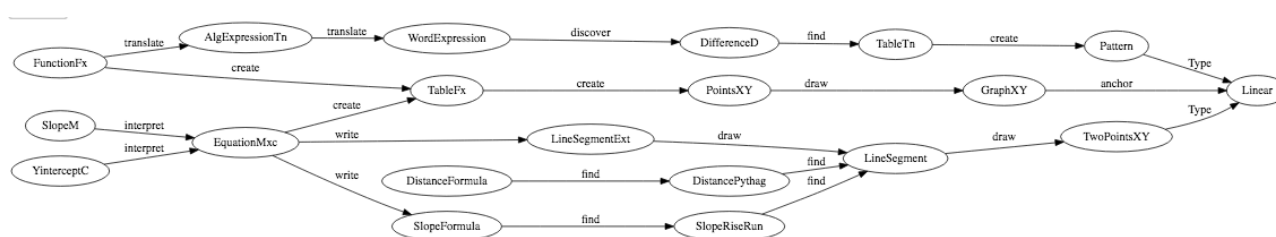
Figure 20: GAM AT Domain Model (Map View)

The arrows used in GAM AT all point back to the overarching concept 'Linear' by default, and this can't be changed.

Methodology and Methods

This research project necessitated an ontological and pedagogical exploration of how mathematical concepts, topics, and learning outcomes can be structured in a domain model for optimal learning (Ahmad et al., 2004; Sosnovsky & Brusilovsky, 2015). Therefore, it was envisaged that a research design that had a qualitative evaluation component would be important.

The evaluation of adaptive learning is a complex research question and it has been suggested that case study methodology is the one best suited to this task (O'Donnell et al., 2015). This is possibly because case studies can be used to achieve a deeper understanding of complex issues (Zainal, 2007). This research project utilises an exploratory case study design that emphasised the characteristics of the domain model, and sought the opinions and subjective accounts of participants (Yin, 2014).

To evaluate the domain models embodied by the various artefacts, it was decided to use a mixed methods research design utilising an online survey and in-depth semi-structured interviews. This approach was used to build *validity* into the research process. It meant it would possible to triangulate the dataset from the survey with the dataset from the interviews, and to possibly make a case for generalising the research findings.

To help the participants in the evaluation process, a screencast was developed to explicate the learning outcome decomposition process, as well as the design and development of the domain model artefacts. There are links to the domain model artefacts and screencast in the 'Supplementary Online Material' section at the end of this paper.

A submission was made to Dublin Institute of Technology's Research Ethics and Integrity Committee (REIC) on 14th November 2017, and approval was granted for this research to be undertaken by email on 6th February 2018.

The participants for the survey were sourced from the Irish Mathematics Teachers' Association (IMTA), and Palmerstown Community School (IMTA, 2018; Palmerstown Community School, 2018). In general, the sources for the survey (IMTA and school) were practicing post-primary mathematics teachers. All of the participants in the online survey were invited to participate via email, using a link to a questionnaire developed using Google Forms (Google, 2018). In general, the questions were constructed using three and five point Likert-type scales. The participants were also provided with web links to the domain model artefacts and screencast, and were asked to evaluate aspects of both.

The school survey (n=10) consisting of 30 questions took place between 27th February and 3rd March 2018. The questionnaire used for the IMTA survey was amended to include an additional six questions. The IMTA respondents (n=26) submitted their responses through Google Forms between 5th March and 16th April 2018. Microsoft Excel was used to merge the data from the two surveys.

A number of *key informants*, who were Mathematics Education Professionals (MEPs) (n=7) and Adaptive Learning Experts (ALEs) (n=2), were invited by email to participate in semi-structured interviews as part of this research study (Table 1). Eight of the interviews took place between 15th March and 12th April 2018, with the final interview occurring on 10th June 2018. Eight of the nine key informants had previously worked as post-primary mathematics teachers. The participants were interviewed individually after they had sufficient time to view the screencasts and interact with the domain model artefacts.

Table 1: Sources for Key Informants used in the Interviews

KEY INFORMANTS	
CATEGORY 1 – MATHEMATICS EDUCATION PROFESSIONALS (MEPs)	**CATEGORY 2 – ADAPTIVE LEARNING EXPERTS (ALEs)**
Project Maths Development Team (PDMT) *	ADAPT Centre, Trinity College Dublin (TCD)
Department of Education and Skills (DES) Inspectorate (Mathematics)	Realizeit Learning, Dublin
National Council for Curriculum and Assessment (NCCA)	
State Examinations Commission (SEC)	
School of Mathematics and Statistics, University College Dublin (UCD)	
Maths Textbook Author 1	
Maths Textbook Author 2	

* Note: **Project Maths Development Team** was rebranded as the **Maths Development Team** in June 2016. However, the acronym PMDT is still used, including on the team's website.

In relation to the seven Mathematics Education Professionals and the two Adaptive Learning Experts, the following random pseudonyms were generated: Martin, Brian, Caoimhe, Sebastian, Vera, John, Maura (MEPs) and Fergus, Walter (ALEs) (RandomNames.com, 2018).

To recap, a domain model is a semantic or hierarchical structure of concepts and sub-concepts, and the relationships between them (Aroyo et al., 2004; Ahmad et al., 2004). Consequently, the dominant themes pervading the survey and interview questions concerned the significance that the participants attached to the teaching and learning of mathematics as (i) a hierarchical system of sequenced concepts and (ii) a system of connected or related concepts, and if the domain model artefacts represented this structure.

The interview questions were based on the ones used for the online survey. This was done in order to achieve triangulation between the two datasets. The interviews were expected to reveal why the survey participants gave particular answers to specific questions. The survey and interview questions were organised pre-ordinately into themes, and it was anticipated that sub-themes might also emerge during the Qualitative Data Analysis (QDA).

There are numerous QDA approaches used to elicit understanding from the complex data that can arise from sources such as interviews, focus groups, observations and journals. This research project used the following QDA approach: (a) identify biases and note overall impressions, (b) reduce, organise and code the data, (c) search for patterns and interconnections, (d) map and build themes, (e) build and verify theories, (f) draw conclusions (O'Leary, 2014).

First, overall impressions were noted by listening to the audio of an entire interview and also during the transcription process. Second, the transcripts were organised into the sections used in the survey and semi-structured interviews. Eliminating sections that were beyond the scope of this paper reduced the data from the transcripts. Third, the responses of the key informants were examined question-by-question and theme-by-theme. Patterns and interconnections were searched for during this examination. Finally, the data from the themes and sub-themes were compared and analysed, and used to produce the results and draw conclusions.

Survey Results
There were two sources for the results presented in this paper. The first source was the data from the online survey of mathematics teachers (n=36), and the second source was the data arising from transcripts of the audio interviews with the key informants (n=9). This section presents the survey results in six tables under the headings of the same six themes in the interview results.

Theme 1: Understanding the Learning Outcome Unpacking Process
The survey participants were asked if they understood the unpacking process from 8 to 45 Learning Outcomes having watched the screencast (Table 2).

Table 2: Understanding the Learning Outcome Unpacking Process after Screencast 2 of 3

	Agree	Neutral	Disagree
Frequency	34	2	0
Percentage	94%	6%	0%

Theme 2: Understanding of Adaptive Learning and Domain Model
The teachers surveyed were asked if they understood the concepts of Adaptive Learning and Domain Model before and after watching the screencast (Table 3).

Table 3: Understanding of Adaptive Learning / Domain Model before and after Screencast 1 of 3

	Before Screencast			After Screencast		
	Agree	Neutral	Disagree	Agree	Neutral	Disagree
Adaptive Learning						
Frequency	4	12	20	24	7	5
Percentage	29%	14%	57%	67%	19%	14%
Domain Model						
Frequency	8	10	18	25	7	4
Percentage	22%	28%	50%	70%	19%	11%

Theme 3: Understanding of the Domain Model Artefacts
The survey participants were asked if they thought that the visual display for the various versions of the domain model artefacts were easy to understand (Table 4).

Table 4: Ease of Understanding of Domain Model Artefacts

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Rhumbl Maps					
Frequency	9	8	7	10	1
Percentage	26%	23%	20%	28%	3%
Mindomo					
Frequency	8	17	4	6	1
Percentage	22%	47%	11%	17%	3%
GAM Authoring Tool					
Frequency	4	18	6	7	1
Percentage	11%	50%	17%	19%	3%

Theme 4: Concept Hierarchy and Sequencing of Concepts
The teachers surveyed were asked if they thought that Junior Cycle Mathematics should be taught and learned as a hierarchical system of sequenced concepts. They were then asked if they thought that the concepts in the various domain model artefacts were correctly sequenced (Table 5).

Table 5: Mathematics should be taught as a Hierarchical System of Sequenced Concepts / Sequencing of Concepts in the Domain Model Artefacts is Correct

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Teach Mathematics as a Hierarchical System of Sequenced Concepts					
Frequency	10	15	8	3	0
Percentage	28%	42%	22%	8%	0%
Sequencing of Concepts in Rhumbl Maps					
Frequency	N/A	N/A	N/A	N/A	N/A
Percentage	N/A	N/A	N/A	N/A	N/A
Sequencing of Concepts in Mindomo					
Frequency	7	21	6	1	1
Percentage	19%	58%	17%	3%	3%
Sequencing of Concepts in GAM Authoring Tool					
Frequency	5	23	6	2	0
Percentage	14%	64%	17%	5%	0%

Theme 5: Relationships between Concepts

The teachers surveyed were asked if they thought that Junior Cycle Mathematics should be taught and learned as a system of connected concepts. In a second question, they were asked if they thought that the concepts in the three domain model artefacts were correctly connected. Finally, the teachers were asked if they thought that mathematics teachers should present 'Patterns and Functions' as a single topic using a unified set of learning outcomes (Table 6).

Table 6: Mathematics should be taught as a System of Connected Concepts /
Connections between Concepts/LOs/Topics in the Domain Model Artefacts are Correct /
Patterns and Functions should be taught as a Single Topic

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Teach Mathematics as a System of Connected Concepts					
Frequency	26	9	0	1	0
Percentage	72%	25%	0%	3%	0%
Connection of LOs/Topics in Rhumbl Maps					
Frequency	7	14	13	2	0
Percentage	19%	39%	36%	6%	0%
Connection of Concepts in Mindomo					
Frequency	6	21	7	2	0
Percentage	17%	58%	19%	6%	0%
Connection of Concepts in GAM Authoring Tool					
Frequency	7	20	8	1	0
Percentage	19%	56%	22%	3%	0%
Teach Patterns and Functions as a Single Topic					
Frequency	17	9	0	0	0
Percentage	65%	35%	0%	0%	0%

Theme 6: Future Adaptive Learning System

The survey participants were asked if they thought that an Adaptive Learning System, with a core Domain Model, could enhance the teaching and learning of Junior Cycle and Leaving Certificate Mathematics. In a follow-up question, they were asked if they believed that an Adaptive Learning System, with a core Domain Model, would be a more effective tool than a textbook for teaching mathematics as a system of connected concepts (Table 7).

Table 7: Adaptive Learning System could Enhance the Teaching & Learning of Mathematics /
Adaptive Learning System would be more Effective than a Textbook

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Enhance the Teaching & Learning of Mathematics					
Frequency	4	16	5	1	0
Percentage	15%	62%	19%	4%	0%
More Effective than a Textbook					
Frequency	5	11	9	1	0
Percentage	19%	42%	35%	4%	0%

Interview Results

In this section, the interview results arising from the transcripts of audio interviews with the nine key informants are presented as six themes.

Theme 1: Understanding the Learning Outcome Unpacking Process

All seven Mathematics Education Professionals (MEPs) said they understood the unpacking process.

The value of unpacking the syllabus LOs was underscored in some of the interviews. John remarked, "every teacher should unpack the learning outcomes…and the connections that exist, whether they are intrinsic or extrinsic, have to be drawn." Caoimhe opined, "I understood the unpacking process and potentially I think that this document will be a valuable resource for teachers in general."

Theme 2: Understanding of Adaptive Learning and Domain Model

Only two and one of the seven MEPs understood the concept of Adaptive Learning and Domain Model respectively prior to the screencast. However, having watched it, this number increased to six MEPs for both concepts.

In relation to Adaptive Learning, both John and Vera said they could see, "what you're trying to do". With reference to the screencast and concept of Domain Model, Brian remarked, "When I went to see what this is all about, it helped me a lot."

One of the ALEs (Walter) outlined a shift in thinking regarding the Domain Model. He said, "The Domain Model, I think people were trying to put too much into it. It became very complex to represent, also maintain." He went on to say, "We're beginning to see more lightweight content models, subject models…"

Theme 3: Understanding of the Domain Model Artefacts

When asked if the Rhumbl Maps artefact was easy to understand, five out of seven MEPs responded affirmatively. The numbers reporting that the Mindomo and GAM AT artefacts could be easily understood were both six out of seven. Sebastian commented about the GAM AT artefact, "I suppose it would help if the arrows went the other way as well." Vera was very enthusiastic about the Rhumbl map views saying, "I think that this is really useful…what I like about this is you get to see the full picture and then you get to zoom in."

Theme 4: Concept Hierarchy and Sequencing of Concepts

Five of the seven MEPs believed that Junior Cycle Mathematics should be taught and learned as a hierarchical system of sequenced concepts. The responses from the MEPs varied from "No" (Vera) to "Yes. Yes. Yes" (Martin) to "This is a very difficult question to answer to tell you the truth" (John).

In relation to the three domain model artefacts, the MEP interviewees were asked if they thought the concepts in each artefact were correctly sequenced. In respect of Mindomo, six out of the seven MEPs responded in the affirmative. In relation to GAM AT, six out of seven said the concepts were correctly sequenced.

Theme 5: Relationships between Concepts

All seven MEPs said that Junior Cycle Mathematics should be taught and learned as a system of connected concepts. Some of the short positive responses from the MEPs were "Yes" (Maura and Sebastian), "Absolutely" (Vera), and "Absolutely as well" (Caoimhe). Brian's more nuanced answer was, "Yes. I'm not sure it can be done but you can strive towards it."

In relation to the three domain model artefacts, the MEPs were asked if they thought the concepts in each artefact were correctly connected. For Mindomo, four out of the six agreed. In relation to GAM AT, all seven stated that the concepts were correctly connected. For Rhumbl Maps, four out of six said the LOs were correctly connected to the topics.

While looking at the GAM AT domain model artefact, Maura remarked, "you would like to think that every teacher's head is like that and the students can make all those connections." The colour scheme used in the Mindomo artefact prompted Brian to exclaim, "I can see lovely connections there... Look. $y = mx + c$, $Tn = dn + a$, $f(x)$...you know" (Figure 17). A number of the MEPs saw the value of connecting topics to learning outcomes in the Rhumbl Maps artefact. Ciaran said, "I think it's useful to click on a topic and to see what learning outcomes are in it." However, Sebastian would have liked the learning outcomes to be connected in Rhumbl. He said, "I think it would be useful if I could click on a learning outcome and see not just what topics are connected but see what other learning outcomes are connected, I think that then would be useful to have."

There was unanimous agreement among all seven MEPs interviewed that teachers should present 'Patterns and Functions' as a single topic using a unified set of learning outcomes.

Theme 6: Future Adaptive Learning System
All seven MEPs interviewed believed that an adaptive learning system could enhance the teaching and learning of Junior Cycle and Leaving Certificate Mathematics.

One of the interviewees implicitly envisaged an adaptive learning system being implemented as part of a blended approach. Martin responded, "With a good teacher and a committed class. I mean the sky's the limit. I think it has huge potential but its implementation and quality would depend on the ability and capability of the teacher."

Six of the seven MEP interviewees were of the opinion that an adaptive learning system would be a more effective tool than a textbook for teaching mathematics as a system of connected concepts. However, Martin opined, "I think the textbook would be superior but if you want to make all these connections or interconnections, I think that a domain model would be highly desirable."

A number of sub-themes emerged during the interviews.

Sub-theme 1: Web
Referring to the GAM AT artefact, John said, "I often describe the syllabus as a web of learning rather than a linear pathway…" In relation to concept hierarchy, Sebastian commented, "So, you would end up much more of a spider web of stuff."

Sub-theme 2: Spiral Curriculum
Sebastian referred to how geometry is taught in tertiary education as a hierarchy of theorems built from axioms and other theorems. However, he proposed a different kind of hierarchy for post-primary mathematics that involved "moving from the concrete up in a spiral kind of a manner I guess. Revisit the same topics through a slightly more abstracted lens..." Caoimhe remarked, "While it's hierarchical, I believe it's the role of the teacher to make explicit connections between what the students are doing so there is a spiral curriculum again." John reflected, "Certainly, there are hierarchies but it's not obvious. It's more of a spiral than a straight line."

Sub-theme 3: Pervasive Concepts
The sub-theme 'Pervasive Concepts' was introduced by John when he said that "There are some gateway ideas which need to be taught as that" followed by "you could teach the entire Junior Cert

Higher Level course with three ideas…"   When Sebastian was asked to rate the importance of teaching Mathematics as a system of connected concepts, he replied, "It's everything. There's only about 10 ideas in Junior Cycle Maths."

Sub-theme 4: Siloed
This sub-theme refers to teaching mathematics as unconnected topics. Sebastian was concerned that Mathematics is "taught in a very 'siloed-off' kind of separate way…" and "there's a danger if you present the syllabus in that way that you're going to encourage the thing to be taught in that way."   John warned, "There are Chinese walls that have been built up over the years between different topics on the course which is dangerous, never mind unhelpful."   Martin suggested, "You might have to do it in discrete parts and then link them up."

Discussion and Analysis
Theme 1: Understanding the Learning Outcome Unpacking Process
When examining whether the screencast enabled participants to understand the LO unpacking process, there was almost complete agreement by all participants that they understood this process having watched the screencast. 94% of teachers surveyed (Table 2), and all of the MEPS interviewed, said they understood the process.

Theme 2: Understanding of Adaptive Learning and Domain Model
Once again the screencast appears to have been very beneficial in helping both sets of participants reach an understanding of the two concepts. Before the screencast, 29% and 22% of teachers said they understood the concepts Adaptive Learning and Domain Model respectively.   These proportions increased to 67% and 70% respectively, having watched the screencast (Table 3). Six out of seven (86%) MEPs acknowledged understanding post-screencast. Maura remarked, "It's very impressive."

Theme 3: Understanding of the Domain Model Artefacts
The survey participants were asked if they thought that the visual display for the various versions of the domain model artefacts were easy to understand (Table 4). In general, the MEP interviewees found the domain model artefacts easier to understand than the teachers surveyed. For the Rhumbl Maps artefact, 49% of those surveyed indicated ease of understanding versus five out of seven interviewees (72%). The figures for Mindomo were 69% versus 6 out of 7 (86%), and for GAT AT 61% versus 6 out of 7 (86%) (Table 4). The higher proportions for the interviewees can possibly be explained by the fact that any queries they had in relation to these artefacts were answered during the interviews.

Theme 4: Concept Hierarchy and Sequencing of Concepts
70% of teachers surveyed agreed or strongly agreed that mathematics should be taught and learned as a hierarchical system of sequenced concepts (Table 5). A similar proportion of MEPs, five out of seven (72%), agreed with this.

There was consistency in the survey participants' responses to the Mindomo and the GAM Authoring Tool artefacts, with 77% and 78% of participants respectively agreeing or strongly agreeing that the concepts in these domain model artefacts were correctly sequenced (Table 5). This question was not applicable to Rhumbl Maps as learning outcomes and topics, with no sequencing, are used in this artefact.

'Concept Hierarchy' turned out to be one of the more interesting themes in the interviews. There were a variety of perspectives on what exactly constituted a hierarchy. Some of the responses shone a light on MEPs' perspectives on "hierarchy." For example, Caoimhe asked the question: "What's the hierarchy? Do you mean linear first, then quadratic and then exponential?" Shortly afterwards,

she answered her own question by commenting: "I think there is (a hierarchy). I think Maths is very iterative. I think you have to start at your base and you build."

Modern adaptive learning authoring systems can 'ingest' electronic textbooks and courses (as Word, EPUB files, etc.), and automatically extract a hierarchy of chapters, topics, subtopics, etc., for a domain model. Academic papers describing this 'automated acquisition of domain model can be found in the literature (Vrablecova & Simko, 2016; Šimko, 2012). Fergus commented about the relationships between concepts in the Domain Model, "To us the hierarchy is just there as a convenience. It's the other relationships that matter."

Theme 5: Relationships between Concepts
There was almost 100% agreement among survey and interview participants that Junior Cycle Mathematics should be taught and learned as a system of connected concepts. Only one of the survey participants disagreed, while all seven MEPs agreed with this strategy. Interestingly, of the 97% of teachers surveyed who agreed, a large proportion (72%) of these strongly agreed (Table 6).

Once again, there was consistency in how the survey participants responded to the Mindomo and GAM Authoring Tool artefacts, with 75% of them agreeing or strongly agreeing that concepts in each of these artefacts were correctly connected. Learning outcomes and topics, rather than concepts, are connected in Rhumbl Maps. The proportion of respondents who agreed or strongly agreed fell from 75% to 58% for the Rhumbl Maps artefact (Table 6). Approximately 10% more MEPs than survey respondents thought that the connections were more correct for the Mindomo and Rhumbl Maps artefacts. While 75% of the teachers indicated correct connections for GAM AT, 100% of MEPs believed the connections to be correct (Table 6). Brian responded, "100%. Yeah. Definitely. Yeah. The connections are really well put together."

There was unanimous agreement among all participants that 'Patterns and Functions' should be taught as a single topic using a unified set of learning outcomes. The strength of agreement was underscored by the fact that 65% of teachers surveyed strongly agreed with this (Table 6). Ciaran underlined the importance that all participants attached to this idea; "I wouldn't necessarily agree with everything in the way that you've structured but I think that idea of the connections between them is massively important."

Fergus explained that user interfaces for some current adaptive learning authoring systems don't represent direction between concepts. (Sebastian had commented on the arrows in GAM AT not going in both directions).

Theme 6: Future Adaptive Learning System
77% of the survey participants believed that an adaptive learning system could enhance the teaching and learning of Junior Cycle and Leaving Certificate Mathematics (Table 7). All seven MEPs interviewed believed this would be the case.

Most MEPs interviewed thought that an adaptive learning system would be a more effective tool than a textbook for teaching mathematics in a connected way (six out of seven). However, the teachers surveyed were not as convinced, with only 61% in agreement and a sizeable 35% neutral on this subject (Table 7). Walter warned, "It's very difficult to author these kind of systems. They tend to be used by the people who built them." He added, "The authoring tools are way behind."

Sub-theme 1: Spiral Curriculum
Sebastian's reflection on how geometry is taught at post-primary level ('spiral', and 'concrete' to 'abstract') seems to implicitly reference two of Jerome Bruner's theories. The first is Bruner's belief that as a curriculum develops, it should revisit basic ideas (the 'spiral curriculum') (Bruner,

1977). The second is that learning occurs by going through three stages of representation, from 'enactive' (concrete) to 'iconic' (pictoral) to 'symbolic' (abstract) (Bruner, 1964).

Conclusions and Future Work

The domain model embedded in the GAM AT artefact uses a design that emphasises relationships between concepts over concept hierarchy, with no overt topics, only interconnected concepts. Three of the seven MEPs preferred this domain model with two preferring Rhumbl Maps, one preferring Mindomo, and one liking all three. Martin commented, "As a map, or an overview, it is impressive and I would love to see it in practice." John liked the 'web-like' aspect to this domain model and wondered what it would look like when extended to the entire syllabus. He commented, "Now, it's going to become complicated as time goes by but that's inevitable and I think, possibly, welcome in fact."

The Mindomo artefact embodies a domain model arranged as five sequences of concepts, to be taught or learned in a fixed order. This fixed sequencing is an issue with this domain model as it imposes a specific pedagogy on teachers. Sebastian commented, "I'm not sure that it's useful. " He then said, "I wouldn't sequence it that way if I were teaching it. But I'm not saying the way I would sequence it is the right way." Walter warned in his interview, "You've got to be careful." He went on to say, "If you start putting more pedagogical rules in the Domain Model, you end up with a domain model that has an embedded pedagogy."

The domain model embedded in Rhumbl Maps has topics connected to LOs but no explicit concepts. There was awareness in building this artefact that it wasn't really a domain model (a) because it used LOs and (b) because it didn't have interconnected concepts or topics. The lack of interconnectedness and learning pathways were picked up by Sebastian in the interviews, "It doesn't connect learning outcome to learning outcome" and "What it doesn't seem to give you is any kind of pathway through them."

Fergus explained how there are two types of relationships between concepts in the domain models of modern systems. First, there are pre-requisite relationships where knowledge of Concept A is required to understand Concept B. These are 'hand-crafted' at the start. Second, there are relationships that are created dynamically by the system as it begins to be used by a large number of users. These are 'data-driven'. He also pointed out that all relationships should be weighted according to importance. The domain models in this research project didn't delineate these different relationships.

Although there was unanimity among participants that mathematics should be taught and learned as a system of connected concepts (100% of participants), it was not clear how this could or should be achieved. Further research in this area is desirable. As John pointed out:

> I think that the approach that we need to take in the future needs to be different from the approach that we're still taking despite ten years of Project Maths at this stage which is we teach topics which appear not to be interconnected and we teach them as an end in themselves so we get the topic done.

The results from this research project suggest that a future adaptive learning system could support mathematics being taught and learned in a more connected way. There was certainly strong agreement that such a system could enhance the teaching and learning of mathematics (77% of survey respondents, 100% of MEPs). This would suggest that the development of a prototype adaptive learning system for mathematics is something worth considering.

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Supplementary Online Material

Screencasts
Explication of Learning Outcome Decomposition and Domain Model Creation Processes

Screencast (Full) - 41 minutes
http://gerardkilkenny.ie/project-full

Screencast (Part 1) - 7 minutes
http://gerardkilkenny.ie/project-part-1

Screencast (Part 2) - 15 minutes
http://gerardkilkenny.ie/project-part-2

Screencast (Part 3) - 19 minutes
http://gerardkilkenny.ie/project-part-3

Domain Model Artefacts
The Three Artefacts Explored in this Paper

GAM Authoring Tool
http://gerardkilkenny.ie/gam-at

Mindomo Map 2
https://www.mindomo.com/mindmap/map-2-c58887d0cdb14fecae5ea43e96a2b8ce

Rhumbl Maps
https://rhumbl.com/embed/5a63e409be3c0f0010f7a0e7

Syllabus
Draft Specification for Junior Cycle Mathematics
https://www.ncca.ie/media/3164/jcmathematics_draft_specification.pdf

Google Forms Surveys
Survey (Mk2) - SCHOOL
http://gerardkilkenny.ie/pd7a-questionnaire-survey-mk2.pdf

Survey (Mk3) - IMTA
http://gerardkilkenny.ie/pd7a-questionnaire-survey-mk3.pdf

Survey - Data Analysis
http://gerardkilkenny.ie/pd8a-data-analysis-surveys.pdf

Interview Questions
Maths Education Professionals (MEPs)
http://gerardkilkenny.ie/pd7b-interview-maths.pdf

Adaptive Learning Experts (ALEs)
http://gerardkilkenny.ie/pd7b-interview-adaptive.pdf